Optimal Transport explains the Representational Similarities between Letters: A pre-registered EEG Study

Jack E. Taylor[1], Rasmus Sinn[1], Christian J. Fiebach[1,2]

[1]Psychology Department Goethe University Frankfurt, [2]Brain Imaging Center Goethe University Frankfurt

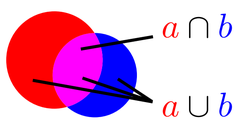
1. Introduction

Orthography is a key bridge between visual input and word recognition. Despite this, perceptual-level processing has often been underspecified in cognitive models.

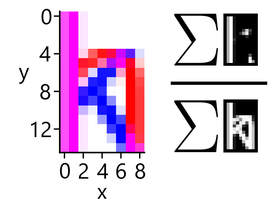
A common approach to measuring similarities of characters' shapes captures the degree of overlap [e.g., 1,2,3]. This approach overlooks global shape information. Optimal transport [4] provides an alternative approach, describing global shape information. We compared how well these approaches predict representational similarities of letters, derived from neural activity.

Jaccard Distance quantifies overlap between two sets, effectively binarising distance.

$$J(a,b) = \frac{|a \cap b|}{|a \cup b|}$$



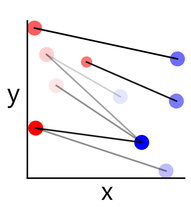
We can use Jaccard Distance between images of letters to capture overlap between key features. However, this *overlooks important relationships between non-overlapping regions*.



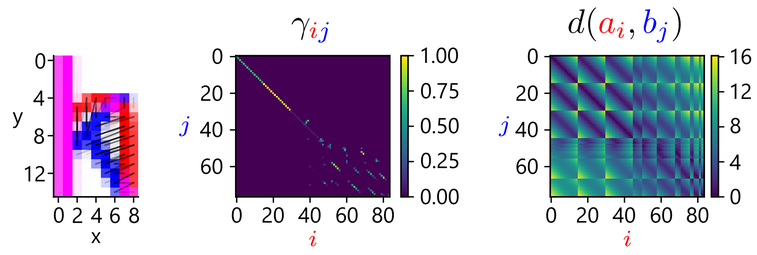
Wasserstein Distance is an optimal transport metric that quantifies a global cost of transporting mass between distributions.

$$W(a,b) = \frac{\sum_{i,j} \gamma_{ij} d(a_i, b_j)}{\sum_{i,j} \gamma_{ij}}$$

Transport Plan *Distance Function*



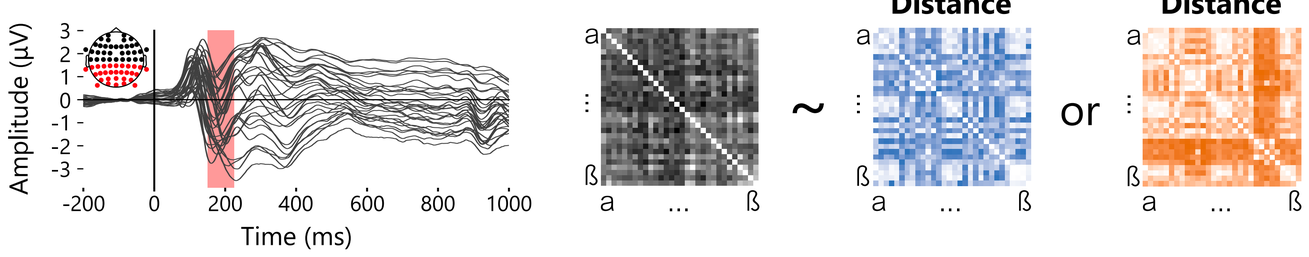
We can calculate Wasserstein Distance between raster images of letters, *capturing global distance between all regions of shapes*.



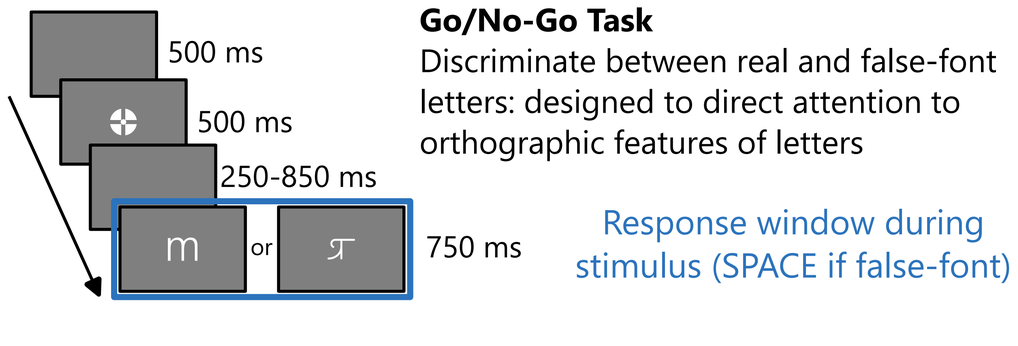
2. Methods

We estimated distances between neural representations for all letters from event- related potentials (ERPs), and then used a representational similarity analysis approach to test how well these representational distances align with Jaccard and Wasserstein distances.

Neural Dissimilarities from ERP Correlation Distances



Preregistered analyses examined rank correlations with neural dissimilarities, calculated from 30 posterior EEG channels, in a 150-225 ms window, for *N*=15 participants..



Go/No-Go Task
Discriminate between real and false-font letters: designed to direct attention to orthographic features of letters

Response window during stimulus (SPACE if false-font)

Arial Light
25 repetitions each

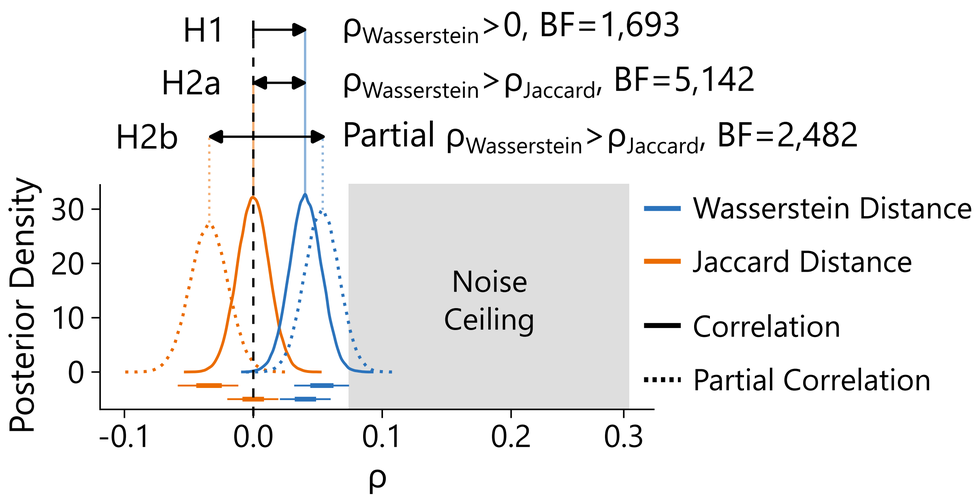
a b c d e f g h i j k l m n o p q r s t u v w x y z ä ö ü ß

BACS2sans
6 repetitions each

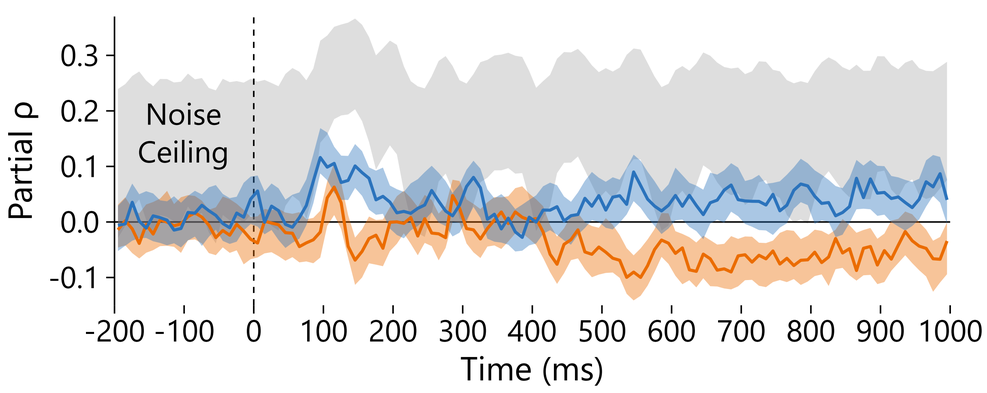
3. Planned Analysis: Posterior N1

Wasserstein Distance and Jaccard Distance were calculated between images of the letters, maximally overlapped via 2D cross-correlation.

Wasserstein Distance between overlap-aligned letters explains their representational similarities, beyond simple overlap in the **150-225 ms period** of interest.

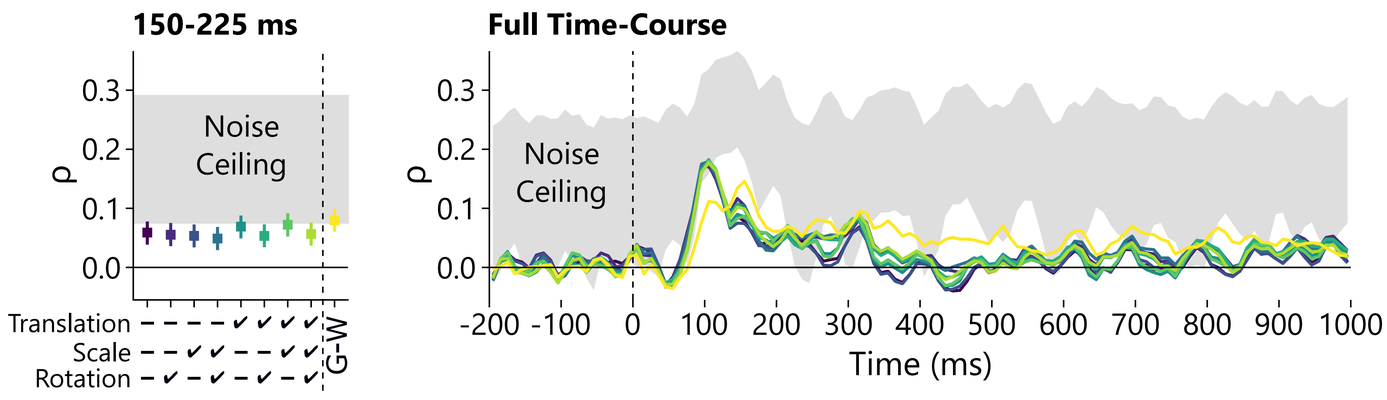


Time-resolved partial correlations show that Wasserstein Distance outperformed Jaccard Distance across the **full time-course**.



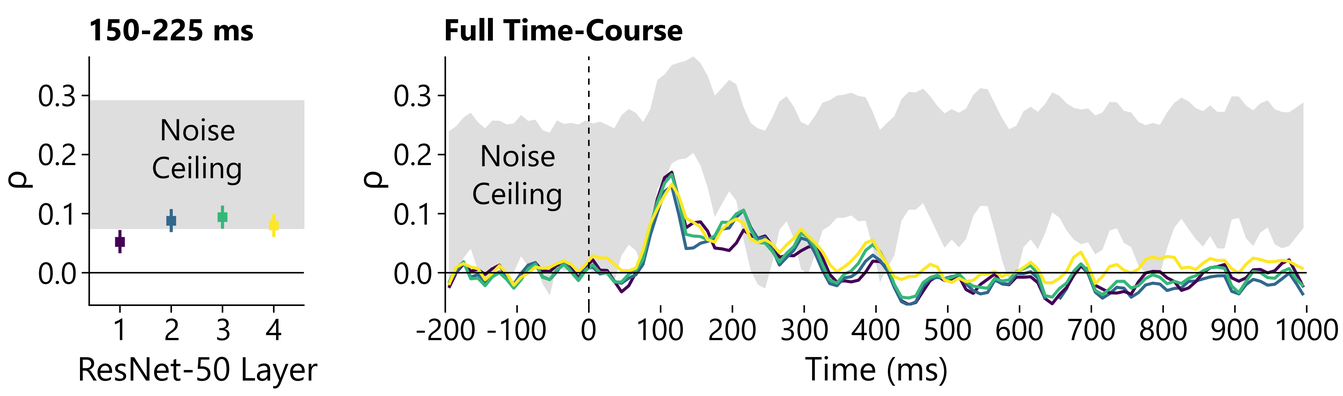
4. Exploratory Analysis: Geometric Invariance

We calculated forms of Wasserstein Distance that are invariant to different geometric transformations. Translation invariance particularly improves performance, and the highly invariant Gromov-Wasserstein (G-W) distance performs best in late periods.



5. Exploratory Analysis: Comparison to ANNs

Artificial Neural Networks (ANNs) are often good predictors of representational similarity, including in visual word recognition [e.g., 5]. We compared Wasserstein Distance to ANNs: its best performing variants perform comparably to the best performing layers of ResNet-50, trained on ImageNet + letters.



6. Summary

Wasserstein Distance provides a parsimonious explanation of the representational similarities of letters, and this cannot be accounted for by simple shape overlap. Wasserstein Distance is a simple and interpretable description, yet it performs similarly to ANNs that require computationally intensive training. These results lay a foundation for parsimonious measures of letter shape similarity, which may be integrated into more complete models of word recognition.

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[2] Fischer-Baum, S., Bruggemann, D., Gallego, I. F., Li, D. S. P., & Tamez, E., R. (2017). Decoding levels of representation in reading: A representational similarity approach. *Cortex*, *90*, 88-102. doi:10.1016/j.cortex.2017.02.017
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