A Multicriteria Decision Model to Compute Optimal Treatment Packages Under Constraint Conditions

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Abstract

A decision model is developed to compute several optimal treatment packages under considerations of treatment compatibility and personal constraints. The mathematical formulation is posed as a minimax problem using a novel application of the entropy principle paired with fuzzy logic to compute optimistic and conservative bounds on the overall effectiveness of a treatment package. The admissibility of fuzzy criteria ratings to describe a patient’s problem provides health professionals with greater flexibility and more confidence in the resulting treatment package. The procedure is general in nature and can be applied to any situation in which an assignment to different options is made on the basis of qualitative or quantitative variables.

Key words: decision theory, fuzzy logic, entropy principle, optimization, medical decision making, compatibility, constraints.

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Introduction

In the past two decades, increasing requirements by psychometricians and health professionals has led to the introduction of a variety of decision models, demonstrated by Petersen (1976), Gilgun (1988), Maas and Wakker (1994), and Vos (1997). Often, decisions concerning psychological and mental health problems involve more complex and subjective assessment criteria. Moreover, mental health professionals often reject the idea of one single treatment solution. Several studies (Greenes, 1986; Lynn & DeGrazia, 1991; Riegelman & Schroth, 1993; Simpson, 1994) have applied decision analysis methods to the health care environment. However, these studies have been limited to multidisciplinary decision making with either a single treatment outcome, or multiple treatment alternatives based on their individual merits. Thus, these approaches may miss the optimal treatment combination because the compatibility issues can only be included when optimizing all treatments simultaneously. Hence, there is a quest for multicriteria decision models capable of handling uncertain criteria ratings and evaluating different treatment alternatives in combination to form a coherent treatment package.

In the development of such decision models several issues arise. The first one is to determine the form of the aggregate estimate against which each decision alternative is evaluated. In most cases, the aggregate estimate is based on some utility function such as the weighted sum of criteria scores. This approach works well when the best single alternative and general ranking of alternatives are sought, as discussed by French (1986). However, this approach will not be of use if an optimal package, containing a minimum number of different alternatives to treat a particular problem is desired. This is because many aggregate estimators mask weak alternatives with strong ones when combining them to compute an overall package effectiveness.

Furthermore, in developing these models, due consideration must be given to treat the criteria ratings as ranges rather than crisp values. While some evaluation criteria can be precisely measured, others are less clearly defined or of subjective nature. Therefore, using ranges of values not only makes the result more accurate but also gives more flexibility to the health professional in rating a patient, and more confidence in the results.

Another problem with existing decision models is that the utility of a treatment is based on the relative distances of the rated criteria to some utility threshold factor. The issues here are that the utility function produces crisp values and also the utility threshold factors are not standardized.
Expressing these single-value utility functions as fuzzy functions would incorporate the varying professional opinions on threshold factors and hence could solve the aforementioned issues. For example, the threshold factor for recommending hospitalization based on the anorexic individual's weight vary significantly. Williamson (1990) makes a strong recommendation for inpatient treatment for the 15% underweight individual. Thompson (1993) suggests a criterion of more than 25% below normal weight, while Beumont, Burrows, and Casper (1987) uses a threshold of 30%. The present paper develops a novel approach to multicriteria decision making to address all of the above limitations. The first one is treated by using a minimax formulation together with the entropy principle, discussed by Hwang and Yoon (1981), which simultaneously involves all participating alternatives to find their respective performances for all criteria in relation to each other. It is worth noting that the combination of treatments may lead to synergistic effects. For example, negative synergistic effects may arise when there is incompatibility between some treatments which could cause adverse effects in patients. A positive synergy exists if the combined treatments have an additional positive effect from mutual causal influences, which is not present in any of the single treatments. Note that these synergistic effects may, in some cases, only be realized when administering the treatments in a sequential manner under time constraints. A salient feature of the proposed model is that negative synergistic effects are prevented when compiling a treatment package through the use of a treatment compatibility matrix. Clearly, this will produce treatment packages very different in content from those assembled by assessing each treatment solely based on its own merit.

The remaining limitations are overcome by using fuzzy logic as discussed, for example, by Athanassopoulos and Podinovski (1997), Kickert (1978), Lootsma (1997) and Negoita (1985). Fuzzy logic incorporates the non-crisp ratings of criteria, and the varying threshold factors into the model with the advantage that upper and lower bounds, representing optimistic and conservative views, can be given for the performance of a treatment package.

Finally, treatment packages optimized under medical considerations only, may not be fully effective unless a patient's personal constraints are also considered. The proposed decision model addresses this aspect in a rigorous analytical manner and computes the best possible treatment package under these conditions.
The Mathematical Decision Problem

A prerequisite to multicriteria decision-making is to understand the von Neumann-Morgenstern (von Neumann & Morgenstern, 1944) normative dictum: maximization of expected utility. Expanded and translated to our requirements it states: find the optimal combination of treatments that minimizes the number of treatments in a package and, at the same time, maximizes the total effectiveness under a number of external constraints.

To achieve this, consider the set of evaluation criteria \( D = \{D_1, D_2, \ldots, D_N\} \) and the set of treatment alternatives \( T = \{T_1, T_2, \ldots, T_M\} \) where \( N \) and \( M \) are the number of criteria and treatments, respectively. Let the numerical values of the criteria \( D_i \) for a patient be described by \( \{d(D_1), d(D_2), \ldots, d(D_N)\} = \{d_1, d_2, \ldots, d_N\} \). These criteria are discussed in more detail in the following section. Let the relations between the proposed treatments and the evaluation criteria be given by \( N \times M \) treatment performance functions \( U_{ij}(d_i), i=1,2,\ldots,N, j=1,2,\ldots,M \). Cases may arise where not all of the most promising treatments can be assembled in one package since some of them may be incompatible and others may be redundant. Let \( C_{ij}(T_i, T_j), i,j=1,2,\ldots,M \) denote the treatment compatibility between the treatments \( T_i \) and \( T_j \). Finally, let there be also a set of personal constraints \( P_k(T_j), k=1,2,\ldots,K \), such as religious and ethnic particularities, environmental, dietary, and socioeconomic considerations where \( K \) denotes the number of constraints per treatment \( j \).

With this, the mathematical formulation can be stated as a minimax problem:

**Lemma 1**: For all criteria \( i \in N \) and all combinations of treatments \( j \in M \), find the \( \mu \) smallest treatment packages of equal effectiveness such that

\[
\mathbf{t}_\mu \in T | \sum_{i=1}^{N} \max_{j \in M} w_j U_{ij}(d_i) = \text{max}
\]

subject to the compatibility constraint \( C_{ij} \) as

\[
\forall T_i, \forall T_j \in \mathbf{t}, C_{ij}(T_i, T_j) = [0,1], i \in N, j \in M
\]

(1)

where the upper limit of the interval \([0,1]\) denotes full compatibility and the personal constraints

\[
\forall T_j \in \mathbf{t}, \forall k P_k(T_j) = [0,1], j \in M, k \in K
\]

(2)

where the upper limit of the interval \([0,1]\) represents total treatment rejection. \( w_i^0 \) denotes the weighting factors discussed in a subsequent section.
The optimization problem given in Lemma 1 is very difficult to solve since not only the effectiveness of a treatment package must be maximized but also the number of treatments in a package must simultaneously take on a minimum. In addition, there could be $\mu$ alternative treatment packages of equal overall effectiveness resulting from different combinations of treatments to form a package. In a later section we will divide Lemma 1 in two parts for which the solutions are easier to find.

**Fuzzy Evaluation Criteria**

Making correct decisions requires the assembly of all pertinent criteria describing the problem. For example, when selecting a treatment for an anorexic patient, various physical and psychological criteria, as well as personal constraints must be considered to ensure treatment effectiveness with the least possible intrusion. Some of these evaluation criteria $D_i$, such as body mass, can be precisely measured and hence can be expressed by a crisp value. Others, such as the role of past abuse in the current mental state of a patient, may be of subjective nature, and can better be expressed by a range of values. In the latter case, using fuzzy ratings will have an important influence on the accuracy of the results, will allow health professionals to express uncertainties and thus will comply with the quest of bounded rationality within cost, time, manpower, material and equipment resources. The advantage of fuzzy logic is that it provides optimistic and conservative results of the problem at hand. The theory of fuzzy logic is given by Lootsma (1997), Negoita (1985) and Kickert (1978).

Let $\left[ d_{i_{\text{min}}}(D_i), d_{i_{\text{max}}}(D_i) \right] = \left[ d_i, \bar{d}_i \right] \in \mathbb{L}$, where $\mathbb{L}$ denotes a measurement scale as, for example, 0 to 10 points, and $d_i$ are the numerical values of the criteria $i$ resulting from the assessment of a patient. The symbol above a rating $d_i$ denotes the minimum or maximum value of a fuzzy criterion rating. Let the values within an interval be equally likely to occur, thus using a uniform distribution. Clearly, the less precise the given ratings are, the wider the interval may become and, in the end, the fuzzier the final result will be. In such a case, if precise data is available, it is easier to narrow the interval of a uniform distribution, rather than to estimate the non-uniform distribution that may govern the values inside the interval.

**Fuzzy Weighting Factors**

It may be the case that not all evaluation criteria are of equal importance. For this reason it is necessary to a priori introduce weighting factors $w_i^\delta$, $i=1,2,\ldots,N$ to lend more
weight to some critical criteria. These weighting factors are also called subjective weighting factors because they show the preferences of an analyst. Let these given weights be normalized such that \( \sum_{i=1}^{N} w_i^s = 1 \).

In addition to the weighting factors \( w_i^s \), there exists another set of weights that are problem-intrinsic and discussed in detail by Chu, Kalaba and Spingarn (1979). These intrinsic weights \( w_i^p \) are dependent on the numerical ratings of the participating criteria in the assessment of alternative treatments. If the ratings of all alternatives for a given criterion \( i \) are equal, then \( w_i^p = 0 \) because this particular criterion is useless for making a decision on the preference of one alternative over another. Contrary, if the ratings are very different, then \( w_i^p \rightarrow 1 \) and this criterion becomes important for the selection process. In other words, the more turbulent or uneven the ratings are, the more important such a criterion becomes in the treatment selection process since the differences between the alternatives are well exposed.

The best measure of such data turbulence is given by the universal entropy principle. Since its discovery, it was principally used in the physical sciences but in recent years it has become an important concept in the social sciences as discussed by Capocelli and De Luca (1973), Hwang and Yoon (1981), and Montroll (1987).

The discrete entropy \( E_i \) is usually stated as

\[
E_i = -k \sum_{j=1}^{M} p_{ij} \ln p_{ij}, \forall i
\]  

where \( p_{ij} \) is the rating of an alternative \( j \) for a criterion \( i \) and \( k \) is a positive constant. Let the utility matrix \( U_{ij} = [U_{\text{min}}, U_{\text{max}}] = [U_{ij}(d_i,d_i), U_{ij}(d_i,d_i)] \) be transformed into a fuzzy utility via the fuzzy criteria ratings \( d_i \). To compute the entropy of the utility matrix \( U_{ij} \), let it be normalized such that

\[
[U_{ij}^*, U_{ij}^*] = [U_{ij}, U_{ij}] \left( \sum_{j=1}^{M} [U_{ij}, U_{ij}] \right)^{-1}, \forall i, \forall j.
\]  

Then, the fuzzy entropy \( E_i \) for each criterion becomes

\[
[E_{\text{min}}, E_{\text{max}}] = [E_i(d_i,d_i), E_i(d_i,d_i)] = - \frac{1}{\ln M} \sum_{j=1}^{M} [U_{ij}^*, U_{ij}^*] \ln [U_{ij}^*, U_{ij}^*], \forall i
\]  

which guarantees that \( 0 \leq E_i \leq 1 \). Note that there will be the case where a rating \( U_{ij}=0 \) leading to \( \ln U_{ij}=-\infty \). However, Equation 5 will still be valid since for
\[
\lim_{U \to 0} U \ln U = \lim_{U \to 0} \frac{\ln U}{U} = -\infty
\]
which, when subject to the rule of l'Hospital yields
\[
\lim_{U \to 0} U \ln U = \lim_{U \to 0} \frac{d(ln U)}{d(U)} = \lim_{U \to 0} U = 0. \text{ Thus, } E(U=0)=0.
\]

If an analyst has no reason to prefer one criterion over another, then the Principle of Insufficient Reason given by Starr and Zeleny (1977) suggests that the best weight distribution is

\[
[w_0^i, w_0^i] = [w_0^p, w_0^p] = \left[1 - \bar{E}_i, 1 - \bar{E}_i\right] \left(\sum_{i=1}^{N} [1 - \bar{E}_i, 1 - \bar{E}_i]\right)^{-1}, \forall i
\]

If, on the other hand, an analyst has given subjective weights \(w_i^s\), then the problem-specific real weighting factors become

\[
[w_i^0, w_i^0] = \frac{w_i^s \left[1 - E_i, 1 - E_i\right]}{1 - \sum_{n=1}^{N} w_i^s \left[E_n, E_n\right]}, \forall i
\]

Following Negoita (1985), the defuzzification of Expression 7 yields two equations for the lower and upper bounds of the weighting factors as

\[
w_i^0 = \frac{w_i^s \min \left[1 - E_i, 1 - E_i\right]}{\max \left[1 - \sum_{n=1}^{N} w_i^s \left[E_n, E_n\right]\right]} = w_i^s \left(1 - \bar{E}_i\right) \left[1 - \sum_{n=1}^{N} w_i^s \bar{E}_n\right]^{-1}
\]

and

\[
\bar{w}_i^0 = w_i^s \left(1 - \bar{E}_i\right) \left[1 - \sum_{n=1}^{N} w_i^s \bar{E}_n\right]^{-1}
\]

where

\[
E_i = -\frac{1}{\ln M} \sum_{j=1}^{M} U_{ij}^* \ln U_{ij}^*, \forall i
\]

and likewise

\[
\bar{E}_i = -\frac{1}{\ln M} \sum_{j=1}^{M} U_{ij}^* \ln U_{ij}^*, \forall i
\]
The advantage of using these weighting factors is that the utility values $U_{ij}(d_i)$ are bound together such that a change in one utility value will have an effect on all others. In this way, the treatment selection process receives a higher resolution since the intrinsic weighting factors are redistributed to lend more importance to those criteria for which the treatments show large differences.

Utility Functions

The elements $U_{ij}$, $i=1,2,...,N$, $j=1,2,...,M$ describe the performances or utilities of $M$ treatments $T_j$ as a function of $N$ criteria $D_i$ that are relevant in treatment selection. In general, a function $U_{ij}(d_i)$ will be nonlinear with respect to the assessed severity $d_i$ of a criterion $D_i$, thus giving rise to complex functional relationships $U_{ij}(d_i)$ as shown in Figure 1, for example.

![Figure 1](image-url)

**Figure 1.** Utility of day treatment as a function of severity of weight loss in anorexia nervosa patients.

Figure 1 describes the utility of day treatment as a function of severity of weight loss in anorexia nervosa patients. The optimistic bound $\text{sup}(U)$ reaches the utility value $U_{ij}=1$ between $d_i = 12\%$ and $d_i = 22\%$; while the conservative bound $\text{inf}(U)$ indicates that the utility $U_{ij}$ will not exceed the value of 0.75. The extent of the discordant expert opinions are represented by the large range of uncertainty between the minimum and maximum bounds $\text{sup}(U)$ and the $\text{inf}(U)$. As the weight loss becomes more serious, day treatment becomes increasingly less recommended in lieu of hospitalization.

It should be pointed out that for every type of problem $N \times M$ functions $U_{ij}$, $i \in N$, $j \in M$ must be constructed, where $N$ and $M$ are the number of criteria and number of treatments,
respectively. Once in a database, these utility functions can be applied to all cases of that particular problem.

**Treatment Package**

With above, the minimax problem given in Lemma 1 can now be restated in a fuzzy environment as:

**Lemma 1**: For all criteria \( i \in N \) and all combinations of treatments \( j \in M \), find the \( \mu \) smallest treatment packages of equal effectiveness

\[
\left[ \mathbf{t}, \mathbf{t} \right]_{\mu} \in T \left\{ \sum_{i=1}^{N} \max_{j \in M} \left[ \mathbf{w}_i^0, \mathbf{w}_i^0 \right], \mathbf{U}_{ij} \left( \mathbf{d}_i, \mathbf{d}_j \right), \mathbf{U}_{ij} \left( \mathbf{d}_i, \mathbf{d}_j \right) \right\} = \max
\]

subject to the compatibility constraint \( C_{ij}(T_i, T_j), \forall i \in N, j \in M \) and the personal constraints

\[
\left[ \mathbf{p}_k, \mathbf{p}_k \right], k \in K.
\]

For easier solution, Lemma 1 can be divided into 2 parts. In the first part, \( \mu \) alternative packages are computed containing compatible treatments that concur to the constraints. Alternative treatment packages may arise, combining different treatments with equal overall effectiveness. This solution, however, may produce large packages containing some treatments of low effectiveness, which would not only be too costly to recommend but would be totally impractical to administer. Hence, the second part of the lemma finds the minimum number of treatments within a given overall package effectiveness. Thus, a treatment \( t_j \) is selected by the minimum or maximum utility, the compatibility, and the minimum or maximum personal constraint.

\[
\left[ \mathbf{t}_j, \mathbf{t}_j \right]_{\mu} \in T \rightarrow \forall i \in N \left\{ \max_{j \in M} \left[ \mathbf{w}_i^0, \mathbf{w}_i^0 \right], \mathbf{U}_{ij} \left( \mathbf{d}_i, \mathbf{d}_j \right) \left( 1 - \max_{k \in K} \left[ \mathbf{p}_k(T_j), \mathbf{p}_k(T_j) \right] \right) \right\} \left( \mathbf{C}_{ij}, \mathbf{C}_{ij} \right)
\]

Note, that the compatibility and personal constraints are added to the Expression 13 to facilitate the optimization process. It can be seen that, if a person fully rejects a treatment, or a treatment \( T_i \) is incompatible with treatment \( T_j \), then the right hand side of Expression 13 becomes zero and the particular treatment \( j \) is rejected. A patient may have some reservation about one or the other treatment as, for example, group therapy suggested to an inhibited person. Because of this, a disliked treatment will not work to the fullest capability and has to be downgraded as given in Expression 13.

Assembling all the individual treatments \( \left[ \mathbf{t}_j, \mathbf{t}_j \right] \) into \( \mu \) packages \( \left[ \mathbf{t}, \mathbf{t} \right]_{\mu} \in T \) produces the total effectiveness for a treatment package \( \mu \).
\[ [e,e]_\mu = \sum_{j \in \{t,t\}} \sum_{i=1}^{N} [w_i^0,\bar{w}_i^0] \left[ U_{ij} \bar{U}_{ij} \right] \left[ 1 - \max_{k \in K} [p_k(T_j),\bar{p}_k(T_j)] \right] \left[ c_{ij},\bar{c}_{ij} \right] \] (14)

where \( j \in t \) are the subscripts used as treatment identifiers of the assembled treatments \( t_j \) in \( t \).

The set \([t,t]_\mu\) may also contain multiple entries of one and the same treatment \( T_j \) since there may be the case where such a treatment is best for a number of evaluation criteria. This, of course, is most desired because it does not increase the number of treatments in a package and yet adds to the overall effectiveness.

For computational purposes, it is of advantage to split, i.e., defuzzify, the expressions 13 and 14 to arrive at two equations for the optimistic and conservative treatment packages \([t,\hat{t}]_\mu\). It is interesting to note that the conservative package \( \hat{t}_\mu \) will generally contain more treatments than the optimistic package \( t_\mu \) for the same overall effectiveness. Clearly, if the number of treatments is forced to be equal for both sets, then the package effectiveness of \( t_\mu \) will be lower. Thus,

\[
\left( t_j \right)_\mu \in T \rightarrow \bar{w}_j^0 \max_{j \in M} \left\{ \inf \left( U_{ij}(d_j),\bar{U}_{ij}(d_j) \right) \left( 1 - \max_{k \in K} [p_k(T_j),\bar{p}_k(T_j)] \right) \bar{c}_{ij} \left( T_i,T_j \right) \right\}, \forall i \in N \quad (15)
\]

and

\[
\left( \hat{t}_j \right)_\mu \in T \rightarrow \bar{w}_j^0 \max_{j \in M} \left\{ \sup \left( U_{ij}(d_j),\bar{U}_{ij}(d_j) \right) \left( 1 - \max_{k \in K} [p_k(T_j),\bar{p}_k(T_j)] \right) \bar{c}_{ij} \left( T_i,T_j \right) \right\}, \forall i \in N \quad (16)
\]

Basically, a defuzzification process is straightforward: Extract from a fuzzy expression two equations such that one will compute the overall minimum and the other the overall maximum. However, a complication may occur if the minimum and maximum treatment utilities \( U_{ij} \) and \( \bar{U}_{ij} \) are functionally interconnected at the boundaries. Consider Figure 1 where the lower and upper bounds are formed by parts of \( U_{ij} \) and \( \bar{U}_{ij} \). Hence, to acquire the minimum expression \( t \), for example, care has to be taken to remain at the minimum boundary of a utility by switching between \( U_{ij} \) and \( \bar{U}_{ij} \) when they cross over. For this reason the inferior (inf) function is introduced as

\[
\inf = \begin{cases} \bar{U}(d_i) \text{ for } \bar{U}(d_i) \leq \bar{U}(d_i) \\ \bar{U}(d_i) \text{ for } \bar{U}(d_i) < \bar{U}(d_i) \end{cases}
\]

Likewise, the maximum boundary of a fuzzy utility function could, in general, be traced by the superior (sup) function as
Consider Expression 15 or 16. There may be a case where either expression yields the same effectiveness for different treatments for a given criterion. Let these treatments be assembled in a subset $\tilde{\mathbf{t}}_i^* \in \tilde{\mathbf{t}}_i$ such that

$$\tilde{t}_j^*, \tilde{t}_m^* \in \tilde{\mathbf{t}}_i^* \rightarrow \forall j, m \in M, j \neq m \mid \tilde{t}_j^* - \tilde{t}_m^* \leq \varepsilon$$

where $\varepsilon$ is a small number denoting the difference between two treatments considered as equal (e.g., $\varepsilon = 0.03 = 3\%$ difference). Then, the total effectiveness of each of these treatments for all criteria has to be calculated in order to select the one with the highest return. Thus,

$$\left( \tilde{t}_j^* \right)_\mu \in \tilde{\mathbf{t}}_i \rightarrow \forall j \in \tilde{\mathbf{t}}_i^* \max_{i=1}^{N} \sum_{i=1}^{N} w_i^0 \sup_{\tilde{t}} \left( U_{ij}(\tilde{d}_i) - U_{ij}(\tilde{d}_j) \right) \left( 1 - \max_{k \in K} P_k(\mathbf{T}_j) \right) \bar{C}_{ij} \left( \mathbf{T}_i, \mathbf{T}_j \right)$$

and similarly for the conservative estimates of the utilities

$$\left( \tilde{t}_j^* \right)_\mu \in \tilde{\mathbf{t}}_i \rightarrow \forall j \in \tilde{\mathbf{t}}_i^* \max_{i=1}^{N} \sum_{i=1}^{N} w_i^0 \inf_{\tilde{t}} \left( U_{ij}(\tilde{d}_i) - U_{ij}(\tilde{d}_j) \right) \left( 1 - \max_{k \in K} P_k(\mathbf{T}_j) \right) \bar{C}_{ij} \left( \mathbf{T}_i, \mathbf{T}_j \right).$$

**Optimal Treatment Package**

The treatment package $[\mathbf{t}, \mathbf{t}]_\mu$ produced by Expressions 17 and 18 may not be optimal since there could be too many treatments in the package, rendering it impractical. Also, there could be some treatments which may not add significantly to the package effectiveness and thus are far too costly to be recommended. For these reasons, a final step is needed to minimize the number of treatments in a package and yet achieve maximum effectiveness.

Let $[\tilde{e}_j, \tilde{e}_j]_\mu \in [\mathbf{e}, \mathbf{e}]_\mu$ denote the minimum and maximum effectiveness of the treatment $[\tilde{t}_j, \tilde{t}_j]_\mu \in [\mathbf{t}, \mathbf{t}]_\mu$. Then,

$$\left( \tilde{e}_j \right)_\mu = \sum_{i=1}^{N} w_i^0 \inf \left( U_{ij}(\tilde{d}_i) - U_{ij}(\tilde{d}_j) \right) \left( 1 - \max_{k \in K} P_k(\mathbf{T}_j) \right) \bar{C}_{ij} \left( \mathbf{T}_i, \mathbf{T}_j \right), \forall j \in [\mathbf{t}, \mathbf{t}]$$

and
Using the treatment effectiveness $[e, e]_\mu$, part two of Lemma 1* can be solved by defining:

**Lemma 2**: Find the smallest optimistic set $\mathbf{t}_\mu^{(\alpha)} \in \mathbf{t}_\mu$ or conservative set $\mathbf{t}_\mu^{(\beta)} \in \mathbf{t}_\mu$ such that for all combinations of $t_j \in M$ there exists a minimum number $\alpha$ or $\beta$ for which the summation of all $\alpha$ elements in $e^{(\alpha)}_{\mu} \in \mathbf{e}_\mu$ or all $\beta$ elements in $e^{(\beta)}_{\mu} \in \mathbf{e}_\mu$ is near the acceptable package effectiveness $\varphi$.

This problem can be solved by classical combinatorics but a more efficient method with less computational effort is described below.

Let $\langle e_{\mu} \rangle = \langle e_{(1)}^{(1)}, e_{(2)}^{(2)}, \ldots, e_{(M)}^{(M)} \rangle_{i_{\mu}}$, $\forall j, n, k \in M$ and likewise $\langle e \rangle_{\mu}$ represent ordered sets in descending order of their numerical values where the superscript denotes the ordinal position in the set and the subscript is the treatment identification number. Then the minimum number of treatments $\alpha$ to form a treatment package $\mathbf{t}_\mu^{(\alpha)}$ can be given by:

$$\text{Find } \alpha \mid \forall j \in \langle e \rangle_{\mu} \sum_{i=1}^{\alpha} e_{(i)}^{(1)} \approx \varphi, \ j \in M$$

In other words, sum the effectiveness of treatments starting with the most effective one $e_{(1)}^{(1)}$ until the desired overall effectiveness $\varphi$ is reached. With this, the resulting optimistic set of treatments becomes

$$\langle \mathbf{t} \rangle_{\mu} = \langle \mathbf{t}_{(1)}^{(1)}, \mathbf{t}_{(2)}^{(2)}, \ldots, \mathbf{t}_{(M)}^{(M)} \rangle_{i_{\mu}}, \ j, n, k \in M.$$

The first treatment in the list will be the most important for the treatment of a particular patient. Using the same procedure the set of the lower bound treatments is obtained as

$$\text{Find } \beta \mid \forall j \in \langle e \rangle_{\mu} \sum_{i=1}^{\beta} e_{(i)}^{(1)} \approx \varphi, \ j \in M$$

Thus,

$$\langle \mathbf{t} \rangle_{\mu} = \langle \mathbf{t}_{(1)}^{(1)}, \mathbf{t}_{(2)}^{(2)}, \ldots, \mathbf{t}_{(M)}^{(M)} \rangle_{i_{\mu}}, \ j, n, k \in M.$$

where, in general, $\beta > \alpha$, i.e., there are more treatments in a conservative package of the same desired effectiveness $\varphi$. Clearly, the number of treatments $\alpha$ and $\beta$ in a package
depends on the given package effectiveness $\varphi$. It is desirable that $\varphi$ is as close to 100% as possible but this may produce large $\alpha$ and $\beta$ resulting in packages difficult to administer and defying bounded rationality.

## Applications

Many researchers (e.g., see Greben & Kaplan, 1995) have demonstrated the advantage in using an integrative approach to mental health treatment. These integrative treatments must be individualized to meet the needs of a patient. However, difficulties arise in the design of an individualized treatment package because of the necessity to consider the interplay of several factors surrounding the disorder, which vary widely from person to person. Consequently, a health professional is faced with the difficult task of determining all the criteria pertinent to treatment selection as well as rating a patient on these criteria. Since many of these criteria will be of subjective nature, treatment selection encompasses an element of uncertainty. Moreover, the selection has to be made under diverse constraints such as treatment availability, cost, time and manpower resources as well as patient constraints.

The presented decision model addresses these issues and has wide applications across many areas of health care. In fact it can be used in all those cases where a number of options are evaluated against a set of criteria with the aim to select a single optimal one or a package containing a minimum number of options for a given overall effectiveness. For any application, the presented decision model can be described by the following sequence:

1. Select a number of criteria $D_i$ describing the medical problem at hand.
2. Assign the subjective weighting factors $w^{s}_i$ for each criterion.
3. Select a set of treatment options $T_j$ to be considered.
4. Construct the treatment compatibility matrix $C_{ij}$.
5. Construct the utility matrix $U_{ij}$, linking the criteria with the treatment options.
6. Evaluate patient on the criteria $D_i$ by assigning the values $d_i$.
7. From the patient obtain the personal constraints $P_k(T_j)$, if any.
8. Compute relevant equations.
9. Rank in descending order the treatments from the obtained optimistic and conservative packages according to their effectiveness.
10. Select an overall package effectiveness $\phi$.

11. Compute the minimum number of treatments $\alpha$ or $\beta$ in a package satisfying $\phi$ by using Equations 21 and 23.

12. For the final results, assemble the first $\alpha$ or $\beta$ treatments from the ranked list computed under item 9.

The decision model is demonstrated on the case of Debra W. suffering from anorexia nervosa. This case was studied extensively by Williamson (1990). For clarity purposes, an abbreviated form of the decision process is given below.

1. Thirty criteria $D_i$ were selected including severity of weight loss, presence of substance abuse, and frequency of binge eating.

2. No weighting factors $w_i^x$ were given by Williamson (1990). Thus, all criteria were considered of equal importance.

3. Twenty-five treatment options $T_j$ were considered including hospitalization, individual therapy, and pharmacotherapy.

4. In constructing the treatment compatibility matrix $C_{ij}$, hospitalization and day treatment, for example, were found to be incompatible.

5. $D_i \times T_j$ (i.e., $30 \times 25 = 750$) fuzzy utility functions $U_{ij}$, were constructed linking the criteria with the treatment options. For example see Figure 1.

6. Debra W. was evaluated on the 30 criteria producing the values $d_i$. For example, she was given the fuzzy rating "none - mild" on the criteria "presence of suicidal tendencies" and the crisp rating of 20% underweight on the criteria "severity of weight loss".

7. The personal constraints $P_k(T_j)$, for Debra W. were not stated in Williamson (1990).

8. All relevant equations were computed.

9. The following optimistic treatment packages were obtained in which treatments are ranked in order of effectiveness:

$$\{t\}_1 = \{\text{hospitalization, cognitive-analytical therapy, reinforcement, family/marital therapy, psychodynamic therapy}\}$$

$$\{t\}_2 = \{\text{day treatment, cognitive-analytical therapy, reinforcement, pharmacotherapy, psychodynamic therapy, nutritional counseling}\}$$
\( \{t\}_1 = \{\text{cognitive-analytical therapy, individual therapy, reinforcement, pharmacotherapy, psychodynamic therapy, nutritional counseling, bedrest}\} \), and likewise the conservative treatment packages:

\( \{t\}_1 = \{\text{hospitalization, cognitive-analytical therapy, reinforcement, family/marital therapy, psychodynamic therapy, individual therapy, parental hyperalimentation}\} \)

\( \{t\}_2 = \{\text{day treatment, cognitive-analytical therapy, reinforcement, pharmacotherapy, psychodynamic therapy, nutritional counseling, bedrest, operant conditioning}\} \)

\( \{t\}_3 = \{\text{cognitive-analytical therapy, individual therapy, reinforcement, pharmacotherapy, psychodynamic therapy, nutritional counseling, bedrest, operant conditioning}\} \),

10. The overall package effectiveness \( \varphi \) was selected as 0.85.

11. Equations 21 and 23 were computed and the minimum number of treatments was determined as \( \alpha = 3 \) for the first treatment package, and \( \alpha = 4 \) for the other two. For the conservative packages, \( \beta = 4 \) for the first treatment package, and \( \beta = 5 \) for the two others.

12. Using \( \alpha \) and \( \beta \), the final treatment packages are assembled. All packages are of equal effectiveness \( \varphi = 0.85 \). The optimistic treatment packages are:

\( \{t\}_1 = \{\text{hospitalization, cognitive-analytical therapy, reinforcement}\} \)

\( \{t\}_2 = \{\text{day treatment, cognitive-analytical therapy, reinforcement, pharmacotherapy}\} \)

\( \{t\}_3 = \{\text{cognitive-analytical therapy, individual therapy, reinforcement, pharmacotherapy}\} \),

and the conservative treatment packages are:

\( \{t\}_1 = \{\text{hospitalization, cognitive-analytical therapy, reinforcement, family/marital therapy}\} \)

\( \{t\}_2 = \{\text{day treatment, cognitive-analytical therapy, reinforcement, pharmacotherapy, psychodynamic therapy}\} \)

\( \{t\}_3 = \{\text{cognitive-analytical therapy, individual therapy, reinforcement, pharmacotherapy, psychodynamic therapy}\} \).

Williamson (1990) arrived at the following treatment recommendation:
While all of his recommended treatments are represented in one or another of the computed treatment packages, they do not all figure in one single package. This stems from the fact that it was impossible for the practitioner to accurately evaluate the joint effectiveness of a treatment package and the treatment compatibility. Furthermore, the optimal treatment packages are composed of considerably less treatment options than recommended by Williamson (1990). The herein-developed multicriteria decision model would suggest that some of the treatments in the latter package are superfluous, and do not add considerably to its effectiveness.

**Conclusion**

A two-dimensional fuzzy optimization process is developed into a decision model to compute optimal treatment packages. The proposed method addresses and solves several areas of concerns inherent to the classical decision methods by using fuzzy arithmetic and the entropy principle. This accounts for uncertainties and doubts in the treatment utilities caused by lack of consensus among health professionals as well as for the uncertainties in the evaluation of patients using subjective criteria.

Furthermore, it has been found that, when computing a treatment package, the compatibility issue between the treatments has to be considered to assure that no negative synergistic effects will arise when applying the selected treatments.

A salient feature of the developed model is that it computes a number of alternative treatment packages of equal effectiveness. This gives a practitioner greater flexibility to select a package that fulfills best the specific needs of a patient.

Finally, the proposed decision model shows that a best possible treatment package for a particular patient may differ from a strictly medical proposal when considering personal constraints that cannot be overcome.
References


