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The Influence of the Initial Associative Strength on the Rescorla-Wagner Predictions: Relative Validity

Makoto Yamaguchi
Waseda University, Japan

The Rescorla-Wagner model's asymptotic predictions are influenced by the initial associative strength under some conditions. This is the case for the experimental paradigm called relative validity. It is possible that stimuli used in the experiment have certain initial associative strength reflecting the subjects' expectancies. Thus researchers must fully understand this influence. In this article, the effects of the initial values on the predictions of the model concerning relative validity are explored. Using a symbolic computation software, these effects were thoroughly revealed.

Keywords: Rescorla-Wagner model, relative validity, causality judgment

One of the most extensively studied topics on classical conditioning is cue competition (e.g., Kamin, 1968; Wagner, Logan, Haberlandt, & Price, 1968), which triggered the influential Rescorla and Wagner's (1972; henceforth R-W) model. Now the model's influence extends to many other areas of psychology including human causality judgment and category learning (see Gluck & Bower, 1988, for example). This recent expansion of influence is due in part to its equivalence to the delta rule, a learning rule employed in connectionist networks, which is derived as method of gradient descent by calculating partial differentiation.

The R-W model describes the updating of associative strength value V. It states that the change in the associative strength of stimulus i, ΔV_i, is determined as follows:

$$\Delta V_i = \alpha_i \beta_{us}(\lambda - \Sigma V_j) \tag{1}$$

Makoto Yamaguchi, Department of Educational Psychology, Waseda University, Tokyo.

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Correspondence concerning this article should be addressed to Makoto Yamaguchi, Department of Educational Psychology, Waseda University, Tokyo 169-8050, Japan.
E-mail: yamag-psy@toki.waseda.jp

where λ is the outcome of a trial and takes the value 100 on reinforced trials and 0 on nonreinforced trials. In some literature, λ takes 1 or 0, but this is not essential. α and β (both > 0) are learning rate parameters; the former reflects the salience of conditional stimuli, and the latter reflects that of unconditional stimuli. Hereafter, the β parameters associated with reinforced trials and with nonreiforced trials are denoted as β_1 and β_2, respectively. Usually it is assumed that $\beta_1 \geq \beta_2$, which seems a natural assumption given the impact of unconditional stimuli for the animals. As indicated by Σ, associative strengths of all stimuli present on a trial are summed. Traditionally, predictions of the model have almost always been derived by computer simulation.

But Yamaguchi (1999) recently showed that recent commercially available software that can perform symbolic computation, such as Mathematica and Maple, is able to solve the equations of the R-W model. Although the model is expressed as a difference equation, the limit can be taken and thus the equation can be considered as a differential equation. The solution is a “closed form” equation in that it is an explicit function of time (or of a trial, in the discrete case).

The purpose of this article is to investigate the effects of the initial values of the associative strength on the R-W predictions for relative validity. This has an important implication particularly for human judgment studies because stimuli used in the experiment may have certain initial values reflecting the subjects’ prior expectancies. Solutions in this article were obtained with Mathematica, but other symbolic computation softwares can similarly be used. In the case of Mathematica, the "DSolve" command solves differential equations. Instead, the “RSolve” command for solving difference equations can also be used. The author has confirmed that both lead to the same conclusion. But in this article the solutions obtained with DSolve command are shown because treating the model as a differential equation will be more suited to deriving ideal predictions that do not involve fluctuation. After obtaining a solution, the “Simplify[%]” command can be used to simplify it. Actual input is not presented in this article, because Yamaguchi (1999) has illustrated many examples and the rule for input construction. Notice that the input examples in Yamaguchi (1999) include such parts as “x[0]==0”, which indicates that x at time 0 equals 0. In contrast, solutions reported in this article were obtained by setting the initial value as a variable instead of 0. In cases where the solution is too long to express in the report, it will be omitted and only asymptotic predictions will be mentioned.

Relative validity

Wagner et al. (1968) found a cue competition phenomenon referred to as relative validity. They used three stimuli (X, Y, Z), and made two compound stimuli (XY and XZ) for animal learning experiments. The former compound was always reinforced while the latter was never reinforced in the experimental condition. This procedure can be concisely represented as ($XY+$, $XZ-$). The target stimulus X was compared to X in the control condition, in which both of these two compound stimuli were reinforced 50% of the time ($XY\pm$, $XZ\pm$). Although X was equally often (50% of the time) reinforced in the two conditions, it evoked much weaker responses in the experimental condition. This experimental paradigm is called relative validity. Later, Wasserman and his colleagues (Van Hamme, Kao, & Wasserman, 1993; Van Hamme & Wasserman, 1993, 1994; Wasserman, 1990) have replicated this phenomenon in a number of experiments using human subjects. Their representative experimental design is to show subjects sheets of papers successively on which information is given about foods a fictional person ate, and about allergy resulting from eating them, and to ask subjects to rate the relationship between foods and allergy.

For this paradigm, Rescorla and Wagner (1972, pp. 81-86) presented extensive explanations for the predictions of their model with all initial values set to 0, and revealed the effects of parameters. Yamaguchi (1999) confirmed them with closed form solutions obtained with a symbolic computation software, and argued that the investigation of the effects of the parameters is very difficult using simulation techniques and thus it is one of the greatest advantages of the closed form solution. Considering the experiments by Wasserman and his colleagues, which listed names of foods on papers a fictional person ate rather than actually presenting real foods to subjects, one can naturally assume salience of the three stimuli to be equal. In general, relative values of α_i do influence the predictions, but it can be ignored here. Therefore, subscript for α is dropped hereafter.

Conclusions by Rescorla and Wagner are: 1) For the experimental condition, the asymptotic value of V_x is 100/3 regardless of the β parameters; 2) For the control condition, it is also 100/3 when $\beta_1 = \beta_2$ and greater than 100/3 when $\beta_1 > \beta_2$. Therefore, the R-W model can accommodate relative validity findings under the assumption of $\beta_1 > \beta_2$. This assumption is intuitively plausible implying that reinforced trials are more salient than nonreinforced trials for animals or humans.

Influence of the initial associative strength

Here, one should bear in mind an important fact about the R-W model: asymptotic values are dependent on initial values for relative validity paradigm (Gallistel, 1990; Melz, Cheng, Holyoak, & Waldmann, 1993). For instance, suppose that all the three stimuli have an initial value of 70. Then asymptotic values are radically changed (see Figure 1 for simulation for the experimental condition), and V_x asymptotes at the value of 10. Positive initial values result in lower asymptotic values for V_x. Some negative initial values (e.g., the three stimuli having the value of –70) even reverse the relative values of V_x and V_y, thus constituting a qualitatively different result.

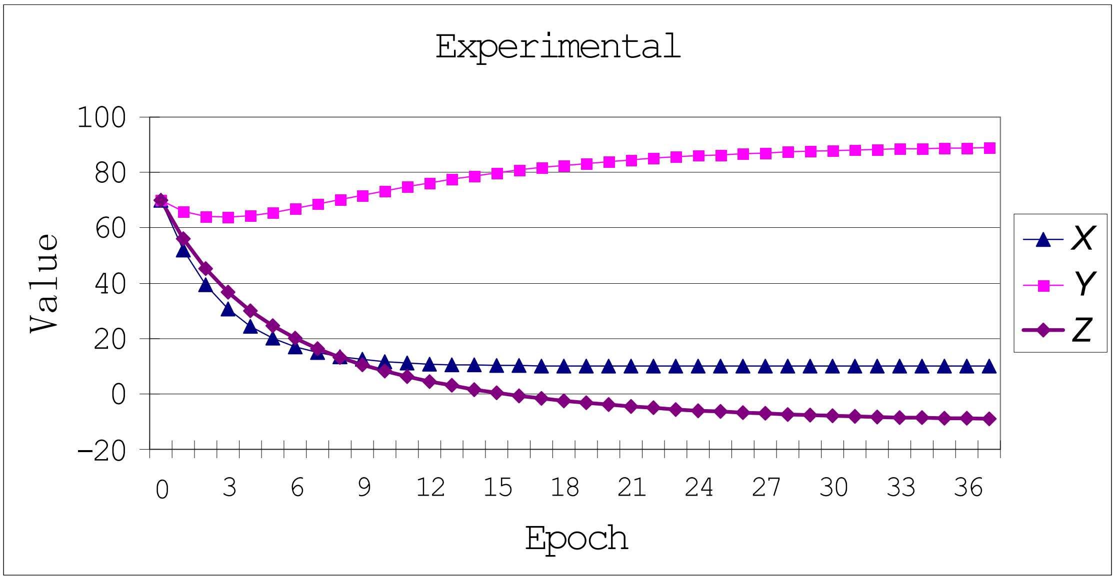

Figure 1. Experimental condition. $\alpha = 0.1$. β parameters are omitted. All the stimuli have the initial value of 70.

Then, one may wonder whether the studies by Wasserman and his colleagues were inappropriate because they did not consider the effects of initial values. Also some of their data suggest that the stimuli might have had some initial positive values: Ratings were initially intermediate values rather than near zero (see Van Hamme & Wasserman, 1994). Given that asymptotic values are radically altered by initial values, were their studies flawed? Fortunately, the present investigation revealed that their studies were not flawed.

The effects of initial values can be clarified by the symbolic computation software. Assume that the initial values of the three stimuli (henceforth *I*'s) are equal. And to begin with, let us assume $\beta_1 = \beta_2$ (thus represented simply by β). As α and β are multiplicative, α*β can be replaced by a new single constant, so one can simply omit β in the input. (As a result, α's below really represent α*β. However, this has no influences on the asymptotic predictions but only affects how fast the asymptote is reached.) The following solution for the experimental condition is revealed by the software:

$$V_x = \{100 - 100\exp(-3\alpha t) + [-1 + 4\exp(-3\alpha t)]I\}/3 \quad (2)$$

It indicates that at asymptote ($t \to \infty$),

$$V_x = (100 - I)/3 \quad (3)$$

For the control condition (see Figure 2), the solution is obtained:

$$V_x = \{100 - 100\exp(-6\alpha t) + [-1 + 4\exp(-6\alpha t)]I\}/3 \quad (4)$$

Thus, the asymptotic value is exactly the same as in Equation 3. Therefore, regardless of the initial values, the asymptote is the same for the two conditions when $\beta_1 = \beta_2$.

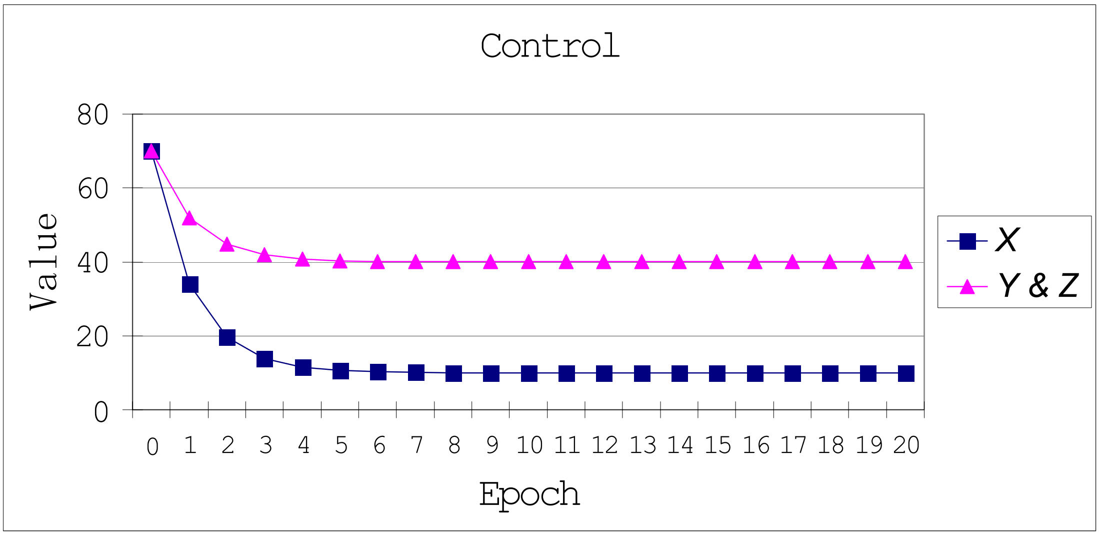

Figure 2. Control condition. α = 0.1. β parameters are omitted. All the stimuli have the initial value of 70.

Next, let us examine the effects of the β parameters to make the exploration more complete. As stated above (also see Melz et al., 1993, for a formal derivation), the β parameters do not affect the asymptotic value in the experimental condition, whereas they alter the asymptotic value in the control condition. See Figures 3 and 4 for simulation, where β_1 is larger than β_2 and all the initial values are 70. Although the experimental condition is different from Figure 1 only preasymptotically, the control condition exhibits asymptotic values different from those in Figure 2.

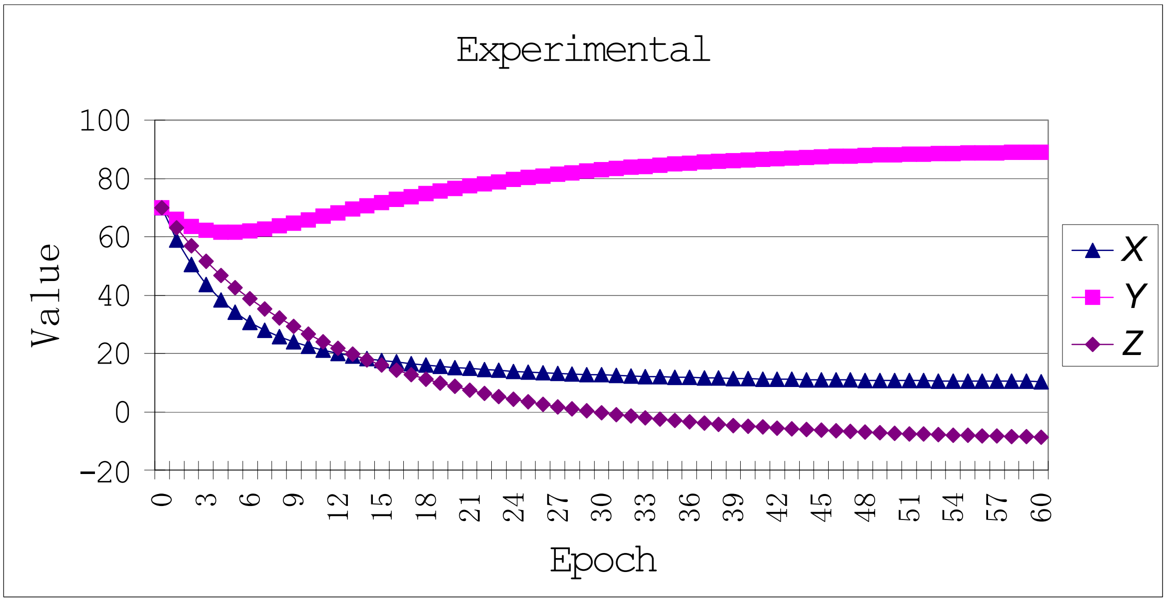

Figure 3. Experimental condition. α parameters are omitted. $\beta_1 = 0.1$, $\beta_2 = 0.05$. All the stimuli have the initial value of 70.

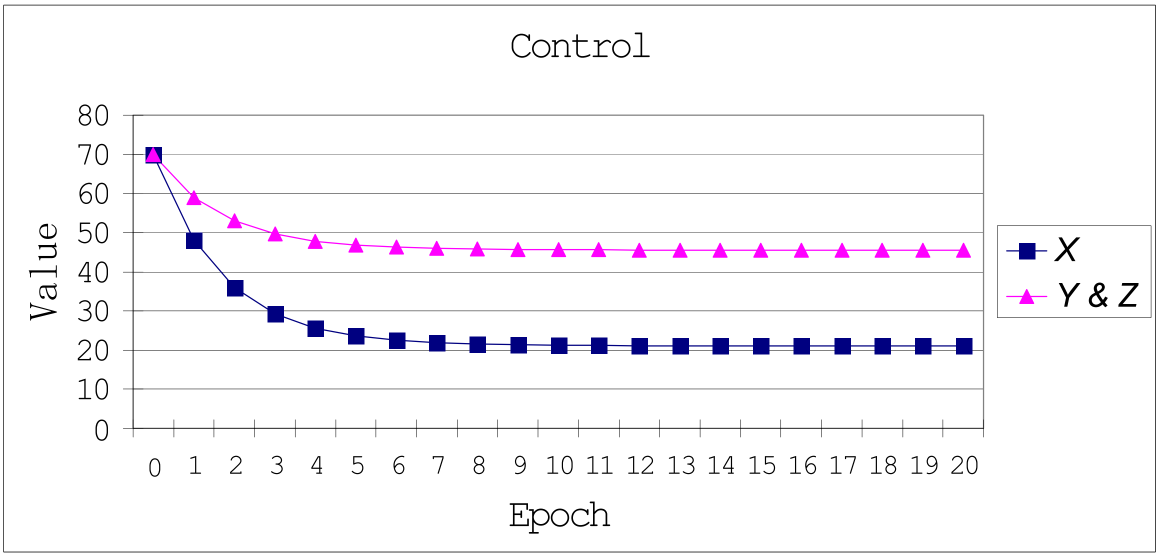

Figure 4. Control condition. α parameters are omitted. $\beta_1 = 0.1$, $\beta_2 = 0.05$. All the stimuli have the initial value of 70.

In both conditions, one can obtain solutions with both β_1 and β_2 treated as variables. Unfortunately, they are interpretable only for the control condition, and the solution for the experimental condition is too long and totally unpractical. But the analytical solution is not necessary for the experimental condition because the β parameters do not affect the asymptote. The solution for the control condition shows that at asymptote,

$$V_x = \frac{200\beta_1 - I(\beta_1 + \beta_2)}{3(\beta_1 + \beta_2)} \tag{5}$$

Now the answer to the problem was obtained. Equation 5 minus Equation 3 indicates that the difference is always positive whenever $\beta_1 > \beta_2$. To confirm this, let us see that the expression reduces to

$$\frac{[200\beta_1 - I(\beta_1 + \beta_2)] - [(100 - I) \cdot (\beta_1 + \beta_2)]}{3(\beta_1 + \beta_2)} = \frac{200\beta_1 - 100(\beta_1 + \beta_2)}{3(\beta_1 + \beta_2)} \tag{6}$$

As the denominator is positive, examine the numerator, which further reduces to $100(\beta_1 - \beta_2)$. Thus, the expression is positive if $\beta_1 > \beta_2$. This conclusion is the same as in the case without initial associative strength.

Conclusion

In sum, regardless of the initial values, the fact remains that the V_x reaches a greater value in the control than in the experimental condition so far as $\beta_1 > \beta_2$. Thus, for the first time, the studies by Wasserman and his colleagues were proved to be valid in this article. But the above exploration concerning the initial values suggests an important implication: future research should explicitly consider the possibility of stimuli having some initial values. Animal learning researchers may be allowed to presume that the stimuli used in an experiment do not initially have associative strength for naïve subjects, but experimenters using human subjects will not be able to preclude such a possibility. Many experiments adopt the cover story with foods, allergy, symptoms or diseases, for instance. Human subjects will have had extensive experiences for such things in real life.

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