Detecting Latent Interaction Effects in Behavioral Data

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Harsh discipline is a well-replicated risk factor for aggressive, antisocial and delinquent behavior. In this paper we investigate the moderator hypothesis that parental perceptions of early child manageability problems moderate parental discipline responses to the child’s disruptive behavior. We describe the application of an interaction model to the analysis of a behavioral data set, show how the data are initially screened for an interaction effect, and implement the model by use of the newly developed Quasi-ML method for analysis of latent interaction effects. The new methodology detects the hypothesized interaction effect by use of a likelihood ratio test. A quantitative interpretation of the estimated model parameters is provided, and conclusions for the analysis of synergistic effects in behavioral data by implementing appropriate interaction hypotheses are drawn.

Keywords: Latent interaction effects, structural equation modeling, aggression, behavioral data.

In the last two decades, structural equation modeling (SEM) has become a common statistical tool for modeling relationships between variables which cannot be observed directly, but only with measurement error. The relationships between these unobservable, latent variables are formulated in structural equations, and they are measured with errors by indicator variables in a measurement model. By the development of software packages for covariance structure analysis such as AMOS (Arbuckle, 1997), EQS (Bentler, 1995; Bentler & Wu, 1993), LISREL (Jöreskog & Sörbom, 1993, 1996a), or

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Mplus (Muthén & Muthén, 1998-2001), SEM has become available to a large community of researchers.

In a social science research context, a linear model structure sometimes provides only a questionable representation of reality. This is particularly the case when in a cross-sectional investigation the size of an effect (e.g., the regressive effect of a certain treatment) itself depends on the outcome of third variables (e.g., socio-economical status, gender, or personality characteristics of the individual). Or in some cases, theory may suggest that the effect of a latent exogenous variable on a latent endogenous variable is itself moderated by a second exogenous variable, thus leading to nonlinear variable relationships.

While ordinary SEM incorporates linear relationships among latent variables, models with nonlinear structural equations have recently attracted increasing attention. Several researchers have called for estimation methods for nonlinear latent variable models, and numerous substantive theories in education and psychology call for analysis of nonlinear models (Ajzen, 1987; Ajzen & Fishbein, 1980; Ajzen & Madden, 1986; Cronbach, 1975; Cronbach & Snow, 1977; Fishbein & Ajzen, 1975; Karasek, 1979; Lusch & Brown, 1996; Snyder & Tanke, 1976). Also, a need for nonlinear extensions of ordinary SEM has been expressed from a methodological perspective (Aiken & West, 1991; Busemeyer & Jones, 1983; Cohen & Cohen, 1975; Jaccard, Turrisi & Wan, 1990), and different ad-hoc estimation approaches have been developed.

Hayduk (1987) established the estimation of an elementary interaction model with one latent product term proposed by Kenny and Judd (1984). Using LISREL 7 (Jöreskog & Sörbom, 1989), they formed products of indicators for measuring the latent product term. Other approaches aim at estimating the nonlinear model within the framework of covariance structure analysis. The technique of forming products of indicators was improved by Jaccard and Wan (1995), Jöreskog and Yang (1996, 1997), and Yang Jonsson (1997) who used nonlinear parameter constraints for estimation of the elementary interaction model under LISREL 8 (Jöreskog & Sörbom, 1996a, 1996b). Simulation studies show that the LISREL-ML estimation procedure can be used for parameter estimation of the elementary interaction model (Yang Jonsson, 1997), but applicability seems to be limited to models with one latent product term because of unstable sampling characteristics of the covariance matrices which include covariances of products of indicators. Moreover, simulation studies for the elementary interaction
model indicate that the LISREL parameter estimators do not have maximum efficiency (Klein & Moosbrugger, 2000; Schermelleh-Engel, Klein & Moosbrugger, 1998).

As an alternative to covariance structure analysis, a two-stage least squares (2SLS) estimation technique has been developed by Bollen (1995, 1996) and Bollen and Paxton (1998) using special linear combinations of indicator variables, so-called instrumental variables, for estimating a quadratic or interaction model with one latent product term. But, although no distributional assumptions are violated for this method, simulation studies showed that 2SLS estimators are substantially less efficient when compared to alternative estimation techniques (Klein & Moosbrugger, 2000; Schermelleh-Engel et al., 1998). Using Bayesian estimation techniques, Arminger and Muthén (1998) proposed a computationally intensive method and demonstrated it for elementary models with one latent product term. Blom and Christoffersson (2001) developed an estimation method based on the empirical characteristic function of the distribution of the indicator variables. But both approaches seem to be limited to elementary models because of their computational burden.

With the LMS (latent moderated structural equations) method, Klein and Moosbrugger (2000) introduced a maximum likelihood estimation technique for latent interaction models with multiple latent product terms. In the LMS method, the latent independent variables and the error variables are assumed to be normally distributed. The distribution of the indicator variables is approximated by a finite mixture distribution and the log-likelihood function is maximized by use of the EM algorithm. In contrast to non-ML estimation methods, LMS also allows for likelihood ratio model difference tests which can test for the significance of one or several nonlinear effects simultaneously. Simulation studies for the elementary interaction model indicate that LMS provides efficient parameter estimators and a reliable model difference test, and they show no indication of bias of standard errors (Klein, 2000; Klein & Moosbrugger, 2000; Schermelleh-Engel et al., 1998). But, although models with multiple latent product terms can be analyzed with the LMS method, this method can become computationally very intensive for models with three or more interacting variables and is sometimes not robust enough when many indicator variables are involved in the measurement model.
In this paper, the newly developed Quasi-ML\(^2\) estimation method (Klein & Muthén, submitted) for structural equation models with multiple interaction and/or quadratic effects is used. This technique has been developed for a more robust and computationally less intensive estimation of complex nonlinear structural equation models, which cannot be analyzed with the LMS method because of the computational burden or occasional robustness problems. The development of an ML estimation procedure for a nonlinear structural equation model is confronted with the fact that the distribution of products of normal variates is non-normal in general, which entails a non-normal multivariate distribution for the indicator variables (Jöreskog & Yang, 1996). The Quasi-ML method used in this paper is based on a variance function model (Carroll, Ruppert & Stefanski, 1995), where conditional mean and variance functions for the nonlinear model are calculated. A quasi-likelihood estimation principle is applied and the density function of the indicator vector is approximated. A Quasi-ML estimator is established by maximizing the log-likelihood function which is based on the approximating density function. Simulation studies show that the Quasi-ML estimates are very close to the Maximum Likelihood estimates given by the LMS method, and the Quasi-ML estimators are almost as efficient as the ML estimators.

The Quasi-ML method is applied to the analysis of a behavioral data set on the relationships between parental perceptions of early child manageability problems, child disruptive behavior, and parental discipline response. A latent variable approach to moderator effects is used to detect and estimate interactive effects and to quantify the moderation of regressive relationships between latent variables.

**Detection of an Interaction Effect in Behavioral Data**

Coercive and harsh parental discipline has been implicated in the eventual development of serious childhood behavior problems (Patterson, Reid, & Dishion, 1992; Snyder & Patterson, 1995; Stoolmiller, 2001). Ample evidence has also accumulated to suggest that children exert evocative effects on parental discipline (Vuchinich, Bank & Patterson, 1992). In this paper we investigate the moderator hypothesis that parental perceptions (measured at boy age 10) of early child manageability problems in the first five years of the boy’s life, moderate parental discipline responses to the child’s disruptive

\(^2\)A demonstration version of the Quasi-ML estimation algorithm can be downloaded under [http://www.ed.uiuc.edu/courses/edpsy494sp03/qml/](http://www.ed.uiuc.edu/courses/edpsy494sp03/qml/)
behavior at age 10. We specifically hypothesize that the more parents have perceived their boys to be difficult to manage even from early on in life, the more aversive will be their reactions to his disruptive behavior. We expect that the regressive relationship between boy’s disruptive behavior (as independent variable) to evoke parental harsh discipline (as dependent variable) is positively moderated by the parent’s current perception of early child manageability problems (as moderator variable).

The data are drawn from the Oregon Youth Study (OYS), two successive Grade 4 cohorts (total $N = 206$, mean age = 10.1) of boys and their families recruited (74% recruitment rate) from schools in the Eugene-Springfield metropolitan area of Oregon serving areas characterized by higher rates of delinquency. The sample is primarily a working class (50%), European American (86%), two-parent sample (70%).

All parent measures were obtained from the primary caregiver, which in 196 families was the mother. In the remaining 10 families, the father was the primary caregiver at the age 10 assessment point. Ratings of early unmanageability were obtained from the parent interview and consisted of a composite of 4 items (strong-willed, over-active, disobedient, and hot-tempered). Both the boy disruptive behavior score and the parental harsh discipline score were based on the second of three, 1-hour occasions (approximately 1 week apart) of naturalistic observation in the family's home at boy age 10. Observers recorded behavior in real time on electronic recording devices to obtain micro-social data and also made global ratings of key behaviors immediately after each 1-hour observation session. For the boy's observed disruptive behavior score two measures were recorded: (a) the proportion (based on frequency) of disruptive behavior and (b) the rate per minute of the same behavior. Parental harsh discipline was assessed with (a) six items from the Home Observer Ratings (e.g., parent's discipline was overly harsh and critical) and (b) a micro-social measure of aversive behavior that was a composite of the proportion (based on frequency) of parental behavior directed to the boy that was aversive and the rate per minute of the same behavior.

**Screening the Data for an Interaction Effect**

The original sample consisted of $N = 206$ observations for the five variables: early manageability problems ($x_1$), proportion measure of boy’s disruptive behavior ($x_2$), rate measure of boy’s disruptive behavior ($x_3$), Home Observer Rating of parental harsh discipline ($y_1$), and micro-social measure of aversive behavior directed to the boy ($y_2$). The
data were initially screened for missing values, outliers, and deviations from normal distribution. 12 cases with missing values were deleted. After that, two cases with extreme univariate outliers for the x-variables (with z-scores exceeding \( \pm 5.3 \)) and three cases with univariate outliers for the y-variables (with z-scores exceeding \( \pm 3.3 \)) were deleted.

The Quasi-ML method has been designed for normally distributed x-variables, but is also known to be robust to moderate deviations of normality. Therefore, skewness and kurtosis were checked for the x-variables. The skewness of the variables \( x_1 \), \( x_2 \), and \( x_3 \) was 0.47, 1.69 and 1.42, respectively; the kurtosis of \( x_1 \), \( x_2 \), and \( x_3 \) was –0.87, 3.50, and 2.49, respectively. A square root transformation was applied to \( x_2 \) and \( x_3 \) in order to reduce deviation from normality. The transformed scores of \( x_2 \) and \( x_3 \) showed a skewness of 0.68 and 0.52, respectively; the kurtosis of the transformed scores was 0.32 and 0.20, respectively.

Finally, the scores were z-standardized for all five variables. The remaining data set used for subsequent analysis had a sample size of \( N = 189 \). The pattern of association among the observed variables was checked by computation of a correlation table (see Table 1).

Table 1.  
Correlation Table for the Five Observed Variables.

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_2 )</td>
<td>.104</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_3 )</td>
<td>.111</td>
<td>.838</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y_1 )</td>
<td>.122</td>
<td>.502</td>
<td>.268</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y_2 )</td>
<td>.116</td>
<td>.467</td>
<td>.333</td>
<td>.871</td>
<td></td>
</tr>
</tbody>
</table>

The correlation table shows that two pairs of indicator variables (\( x_2 \) and \( x_3 \) for boy’s disruptive behavior, and \( y_1 \) and \( y_2 \) for parental harsh discipline) are each highly correlated. The correlations between the indicators \( x_2 \) and \( x_3 \) for boy’s disruptive behavior and the indicators \( y_1 \) and \( y_2 \) for parental harsh discipline range from .27 and .50.

A possible moderating effect of the observed moderator variable “early manageability problems” (\( x_1 \)) on the regressive relationship between boy’s disruptive behavior and parental harsh discipline was first investigated descriptively by a comparison of the regres-
sive relationships across the three terciles of the moderator variable $x_1$. For this purpose, the data set of sample size $N = 189$ was divided into three data sets of $N = 63$ each, representing cases with low, medium, and high levels of early manageability problems. For each of these three terciles, a scatter plot and a regression line for the indicator variables $x_2$ and $y_2$ was produced. The plots are given in Figure 1.

![Data-based regression lines and scatter plots between measures $x_2$ (boy’s disruptive behavior) and $y_2$ (parental harsh discipline), grouped for cases with low, medium, and high level of early manageability problems.]

Figure 1. Data-based regression lines and scatter plots between measures $x_2$ (boy’s disruptive behavior) and $y_2$ (parental harsh discipline), grouped for cases with low, medium, and high level of early manageability problems.

The three graphs display the change of the regressive relationship between boy’s disruptive behavior and parental harsh discipline across three levels of the moderator variable. The scatter plots for the terciles of the moderator variable $x_1$ (early manageability problems) show a clear indication of a moderating effect of $x_1$: The slopes of the regression lines increase with increasing level of early manageability problems across the three terciles of $x_1$. The higher the level of early manageability problems, the higher is the slope of the regressive relationship between boy’s disruptive behavior and parental harsh discipline. Starting from low over medium to high levels of early manageability prob-
lems, the correlation coefficient between $x_2$ and $y_2$ increases from .34 over .46 to .58 across the terciles. The display of the data strongly suggests that early manageability problems and boy’s disruptive behavior interact in their common influence on the evocation of parental harsh discipline: In cases where the level of manageability problems is high, an evocation of parental harsh discipline can be expected when the level of boy’s disruptive behavior is also high.

**Implementation of Interaction Model**

In the framework of latent variable modeling, a latent interaction model was formulated for the analysis of the behavioral data. The structural equation of the model is

$$
\eta = \alpha + \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_1 \xi_2 + \zeta
$$

where $\eta$ (parental harsh discipline) is a latent endogenous variable measured by $y_1$ and $y_2$, $\alpha$ is an intercept, $\xi_1$ (early manageability problems) is an exogenous variable measured by $x_1$, $\xi_2$ (boy’s disruptive behavior) is a latent exogenous variable measured by $x_2$ and $x_3$, and $\zeta$ is a disturbance term. The parameter $\gamma_1$ gives the linear effect of $\xi_1$ on $\eta$, $\gamma_2$ gives the linear effect of $\xi_2$ on $\eta$, and $\gamma_3$ is the interaction parameter which gives the impact of the interaction effect between $\xi_1$ and $\xi_2$ on $\eta$. The term $[\gamma_2 + \gamma_3 \xi_1]$ is the moderator function which gives an asymmetric interpretation of the nonlinear model equation. It represents the fact that the slope of the regressive relationship between $\xi_2$ and $\eta$ changes across levels of the moderator variable $\xi_1$: Depending on the outcome of $\xi_1$, the slope changes due to the value of the moderator function. The measurement of the latent variables is given as a common measurement model by

$$
\begin{align*}
\eta &= \alpha + \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_1 \xi_2 + \zeta \\
\eta &= \alpha + \gamma_1 \xi_1 + [\gamma_2 + \gamma_3 \xi_1] \xi_2 + \zeta
\end{align*}
$$

where $\lambda_x$ and $\lambda_y$ are loadings, and $\delta_2, \delta_3, \varepsilon_1, \varepsilon_2$ are measurement errors. In our case, there is only one indicator variable, $x_1$, available for the measurement of the construct $\xi_1$ (boy’s disruptive behavior). Thus, $\xi_1$ is assumed to be measured without error by $x_1$. In fact, mathematically, the specified model deals with an interaction effect between an observed exogenous variable ($x_1$) and a latent exogenous variable ($\xi_2$). The measurement
equations don’t include intercepts because standardized data are used for analysis. The specified model has 13 free parameters.

The structural equation model was analyzed using the Quasi-ML estimation algorithm (Klein & Muthén, submitted). As opposed to ordinary ML estimation, the Quasi-ML algorithm maximizes a so-called quasi likelihood function (Carroll, Ruppert & Stefanski, 1995) with respect to the model parameters. The maximization of the quasi likelihood function provides an approximate maximum likelihood estimate for the model parameters. The estimation results obtained with the Quasi-ML method for the interaction model are given in the path diagram in Figure 2. The path diagram shows the fully standardized solution.

\[
\eta = -0.019 + 0.08\xi_1 + [0.52 + 0.17\xi_1]\xi_2 + \zeta.
\]

\[\xi_1\] 
Boy’s early manageability problems

\[\xi_2\] 
Boy’s disruptive behavior

\[\eta\] 
Parental harsh discipline

\[x_1\]

1.00

\[x_2\]

0.99

\[x_3\]

0.84

\[\delta_2\]

\[\delta_3\]

\[\zeta\]

0.17

0.11

0.08

0.52

0.95

0.92

\[\varepsilon_1\]

\[\varepsilon_2\]

Figure 2. Path diagram of Quasi-ML method estimation results for the behavioral data (fully standardized solution).
The reliability of the indicator variables was .98 for \( x_2 \), .70 for \( x_3 \), .91 for \( y_1 \), and .84 for \( y_2 \). The estimated moderator function \([\gamma_2 + \gamma_3 \xi_1] = [.52 + .17 \xi_1]\) reflects the fact that the slope of the regression of \( \eta \) (parental harsh discipline) on \( \xi_2 \) (boy’s disruptive behavior) increases with increasing level of \( \xi_1 \) (early manageability problems). The linear effects account for 28% of the variance of the latent endogenous variable \( \eta \), and the interaction effect accounts for an additional 3% of the variance of \( \eta \). But the relevance of the interaction effect becomes clearly visible when, conditional on different levels of the moderator variable \( \xi_1 \), the variance of \( \eta \) accounted for by \( \xi_2 \) is regarded: For example, given a low level of early unmanageability (z-score of \( \xi_1 = -1.5 \)), \( \xi_2 \) accounts only for 7% of the variance of \( \eta \), but given a high level of early unmanageability (z-score of \( \xi_1 = +1.5 \)), \( \xi_2 \) accounts for 60% of the variance of \( \eta \).

The hypothesis of an interaction effect was evaluated by testing whether the interaction parameter \( \gamma_3 \) is significantly different from zero. The Quasi-ML method executes a likelihood ratio test which tests the model with the interaction parameter \( \gamma_3 \) as a free parameter against a model with \( \gamma_3 \) fixed to zero. The test of the interaction effect was significant with \( \chi^2 = 4.89 \) (\( df = 1 \), \( p < 0.03 \)).

Although at present no statistically correct fit statistic which takes the specific non-normal distribution of product terms into account is available, the fit of the model was inspected by computation of the model-implied correlation coefficient between \( x_2 \) and \( y_2 \) across levels of the moderator variable. For the three terciles of the sample, the means for the moderator variable \( x_1 \) are –1.07, -0.16, and +1.19, respectively. Based on the estimation result of the Quasi-ML method and conditioned on these three means, the correlation coefficient between \( x_2 \) and \( y_2 \) for the terciles is .37, .48, and .56, respectively. When these values are compared to the data-based correlations, this illustrates that the interaction model provides a good fit to the conditional correlational structure of the data when conditioned on the moderator \( x_1 \) (early manageability problems).

**Discussion**

The Quasi-ML method detects the hypothesized interaction effect between early unmanageability problems and boy’s disruptive behavior at the 5% Type I error level. Also, it can be noted that the data-based and the estimation-based correlation coefficients between \( x_2 \) (indicating boy’s disruptive behavior) and \( y_2 \) (indicating parental
harsh discipline) for the terciles of the moderator variable $x_1$ (early manageability problems) show a close match. Based on these findings, the specified interaction model with an interaction effect between an observed ($\xi_1 = x_1$) and a latent variable ($\xi_2$) gives an adequate representation of the moderated relationship described for the observed behavioral data. This illustrates the usefulness of the application of an interaction model to explain conditional regressive relationships and the applicability and efficient use of the Quasi-ML method to the analysis of the data.

On the substantive level, the estimation result of the Quasi-ML method strongly supports the research hypothesis that parental perceptions of child early manageability problems increase the risk of harsh discipline for a child in response to everyday disruptive child behavior. Harsh discipline is a well-replicated risk factor for aggressive, antisocial and delinquent behavior. Thus, the moderator effect may represent a pernicious self-fulfilling prophecy for the parent. Because the parent perceives the child to have been difficult, the parent reacts more harshly to everyday misbehavior, which eventually contributes to the child genuinely becoming more difficult to manage. The parental perceptions of early unmanageability could have arisen because the child actually was difficult to manage or the mother may have had unrealistic expectations of the demands of childrearing or both. Regardless of their origin, however, the results here suggest that in combination with everyday misbehavior, they constitute a synergistic risk factor for future antisocial behavior.

References


